國立交通大學 105 學年度第 1 學期
博士班資格考筆試考試試題

土木工程學系 水利組　科目：工程數學　選考學生數：1　考試時間：60 min

1. (25%) Find an orthogonal matrix S and a diagonal matrix D such that $S^{-1}AS = D$.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

2. (25%) Solve the following pde: $\frac{\partial^2 U}{\partial t^2} = 4\frac{\partial^2 U}{\partial x^2}$, $0 \le x \le 1$, $t > 0$

$U(0,t) = U(1,t) = 0$; $U(x,0) = \frac{1}{2}\sin(\pi x) + \frac{1}{4}\sin(3\pi x)$, $\frac{\partial U}{\partial t}(x,0) = 0$

3. (30%) Find the solution for a differential equation

$\frac{d^2 y}{dt^2} + 25y = \cos 3t + \sin 5t$ with initial conditions of $y(0) = 0$ and

$y'(0) = 0$.

4. (20%)

(1) Is the equation $dx + (3x - e^{2y})dy = 0$ exact? Explain! (5%)

(2) Find an integrating factor F(y)? (10%)

(3) Find the solution of this equation. (5%)

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土木工程學系 水利組　科目：流體力學　選考學生數：1　考試時間: 60 min

1. The x directional velocity component in a steady, two dimensional incompressible irrotational flow field is $u = 2xy$. Determine: (a) the y direction velocity component v. (15 %) (b) the stream function and the velocity potential. (10 %)

2. Consider a steady flow of a viscous fluid between two infinity long, parallel, vertical plates, spaced a distance of h apart, Fig. 1. The plate on the left (x = 0) is stationary; the plate on the right (x = h) is moving upward at constant speed W. Assume that the flow is fully developed in the z (or vertical) direction. All pressure gradient are negligible but we can not neglect the effect of gravity, which act in the negative z direction. Find expression for the vertical velocity components w. (25 %)

3. The water supply reservoir shown in Fig. 2 has an average depth 20m, a surface area of 20 km^2, and an outlet whose centerline is 15 m below the water surface. If the outlet diameter is 1 m, what is the outflow and its associated velocity? (15%) What would be the draw downs (drop in water surface elevation) during one-week and one-day periods? (15%)

4. What rules of similitude should be followed in a model test? Explain those rules of similitude. (20%)

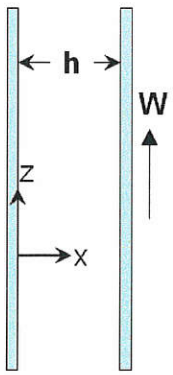

Fig. 1 for problem 2

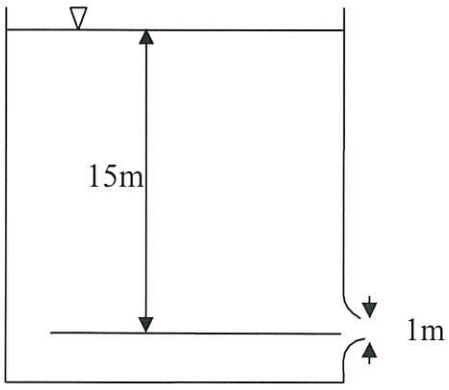

Fig. 2 for problem 3 (Not in scale)

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土木工程學系 水利組　　科目：水資源規劃　　選考學生數：1　　考試時間：60 min

1. Assume that the average monthly evaporation loss and precipitation for several years for a given reservoir site are as follows:

Month	10	11	12	1	2	3	4	5	6	7	8	9
EV (AF)	270	275	280	350	470	450	400	350	370	330	300	290
PP (AF)	3	5	5	10	30	50	100	150	70	1	2	3

The monthly inflows to the reservoir for several years are as follows:

Year	Month	Inflow (AF)	Year	Month	Inflow (AF)
1980	10	742	1982	1	4410
	11	1060		2	2750
	12	1000		3	3370
1981	1	1500		4	5170
	2	1080		5	19680
	3	6460		6	19630
	4	10000		7	3590
	5	13080		8	710
	6	4910		9	518
	7	981		10	924
	8	283		11	1020
	9	322		12	874
	10	404	1983	1	1020
	11	787		2	8640
	12	2100		3	6370

where AF is the volume of acre-ft. Determine the required active storage for the given reservoir to produce 2,000 AF/month firm yield. (30%)

2. Consider the following minimization problem

Minimize $f(\mathbf{x}) = -4x_1 - 6x_2 + 2x_1^2 + 2x_1x_2 + 2x_2^2$

Subject to

$$x_1 + 2x_2 = 2$$

Solve the problem by the Lagrange multiplier method. (20%)

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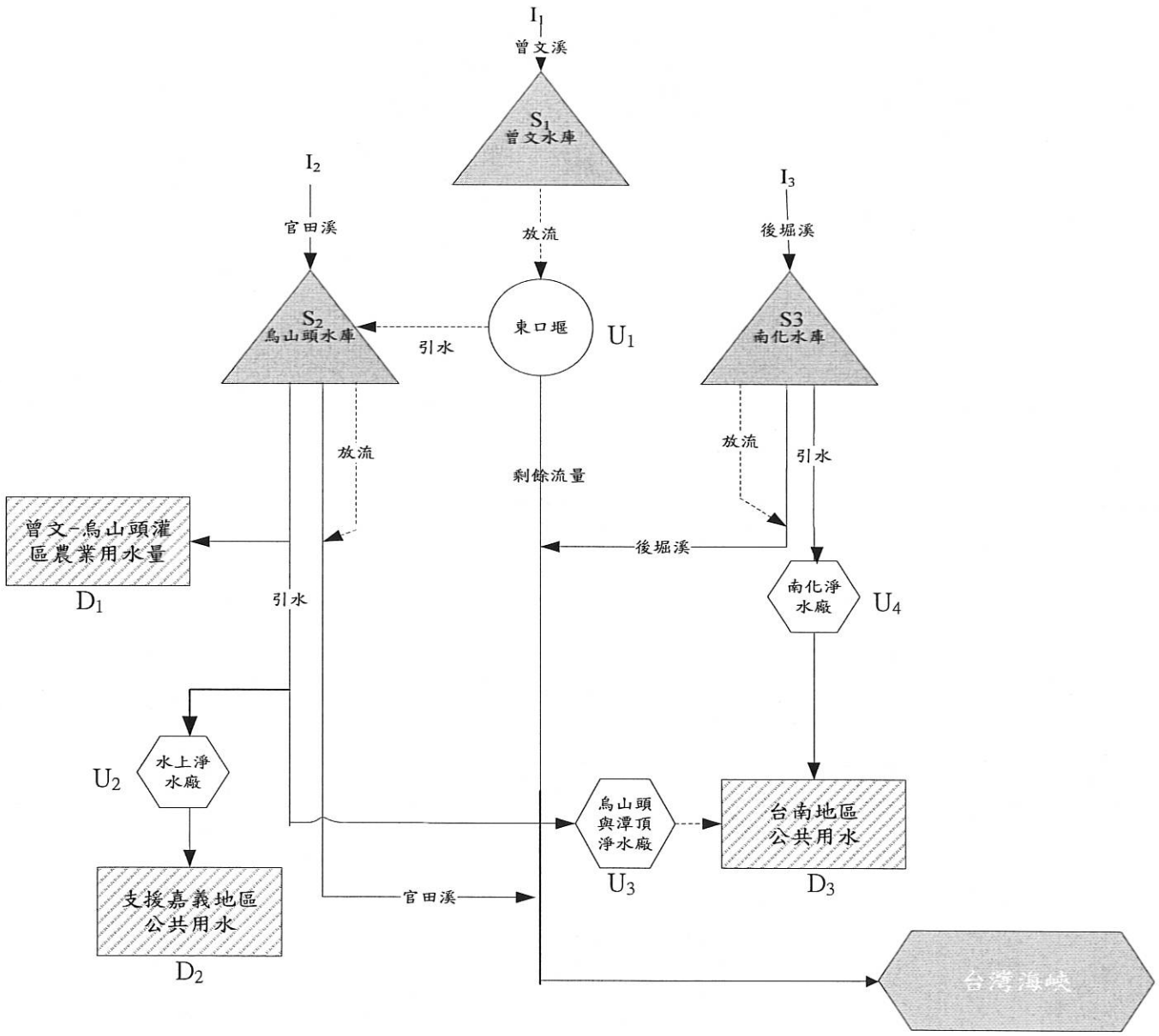
土木工程學系 水利組　　科目：水資源規劃　　選考學生數：1　　考試時間：60 min

3. 下圖 Figure 3 為南部地區簡化後之水庫系統示意圖請寫出此系統之線性規劃水資源調配模式，請自行定義變數，寫出目標函數及應考量之限制式，並說明之。(25%)



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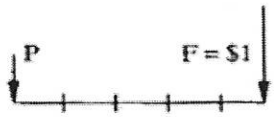
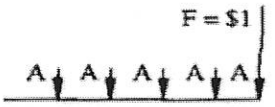
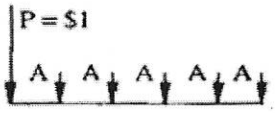
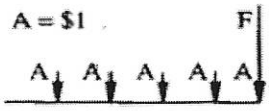
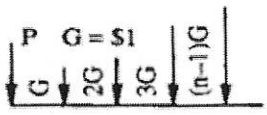
土木工程學系 水利組　　科目：水資源規劃　　選考學生數：1　　考試時間: 60 min

4. 一水資源開發計畫，在第一年結束後的效益為 10,000 元，之後到第五年的效益皆以定額的方式增加，故第五年結束時的利益為 50,000 元，接著利潤持續維持在 50,000 元至第十年末，而第十一年因設施擴張利潤增為 55,000 元，直到三十年末之前利潤皆持續在 55,000 元，接著以定額的方式減少，至第 40 年結束時的利益為 0 元。請問此計劃的效益現值為多少？假設 1~10 年期間利率為 6%，11~40 年期間利率為 5%。(可參考下表不同情況下現值之轉換因子公式)(25%)

TABLE 2.1.1
Summary of discounting factors

Type of Discount Factor	Symbol	Given*	Find	Factor	
Single-Payment Factors					
Compound-amount factor	$\left(\frac{F}{P}, i\%, n\right)$	P	F	$(1+i)^n$	
Present-worth factor	$\left(\frac{P}{F}, i\%, n\right)$	F	P	$\frac{1}{(1+i)^n}$	
Uniform Annual Series Factors					
Sinking-fund factor	$\left(\frac{A}{F}, i\%, n\right)$	F	A	$\frac{i}{(1+i)^n-1}$	
Capital-recovery factor	$\left(\frac{A}{P}, i\%, n\right)$	P	A	$\frac{i(1+i)^n}{(1+i)^n-1}$	
Series compound-amount factor	$\left(\frac{F}{A}, i\%, n\right)$	A	F	$\frac{(1+i)^n-1}{i}$	
Series present-worth factor	$\left(\frac{P}{A}, i\%, n\right)$	A	P	$\frac{(1+i)^n-1}{i(1+i)^n}$	
Uniform Gradient Series Factors					
Uniform gradient series present-worth factor	$\left(\frac{P}{G}, i\%, n\right)$	G	P	$\frac{(1+i)^{n+1}-(1+ni+i)}{i^2(1+i)^n}$	

*The discount factors represent the amount of dollars for the given amounts of one dollar for P, F, A and G.

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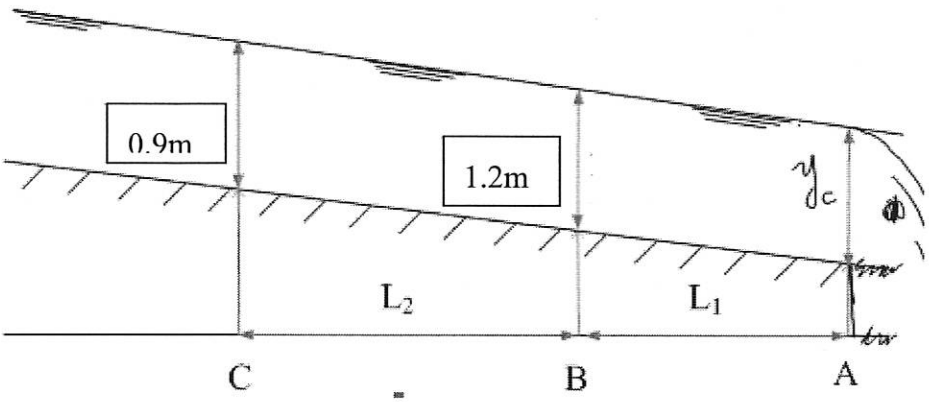
1. A rectangular channel is 3 m in width and draws water from a lake. The channel slope is 0.001 and the Manning *n* is 0.013. What will be the discharge in the channel when the lake level is 2 m above the high point of the channel bottom? Neglect the inlet losses. (25%)

2. Water flows with a velocity of 1.5 m/sec and a depth of 2.5 m in a rectangular channel. Determine (a) the maximum size of rise in the channel bottom without affecting flow condition upstream of the rise in the bottom, and (b) the depth over the rise when the height of the rise is one-half of the height in (a). (25%)

3. 下圖所示為某一矩形渠道之縱剖面示意圖，已知渠道寬為 2 公尺，底床坡降S_0為 0.002、曼寧糙度 n 為 0.012。假設流量為 6 cms，且 A 點之水深 Yc 為臨界水深 (Critical depth)，(1)若圖中 B 點之水深為 1.2 公尺，試計算 AB 兩點間之水平距離 L_1。(2) 若圖中 C 點之水深為 0.9 公尺，試計算 BC 兩點間之水平距離 L_2。(25%)



4. A hydraulic jump occurs in a horizontal rectangular channel with sequent depths y1=0.85m and y2=5.95m. Calculate (1) The Froude number Fr_1 before the jump, (2) The Froude number Fr_2 after the jump, (3) The flow discharge per unit width q and (4) The critical depth. (25%)

$$y_2 = \frac{y_1}{2}\left[-1+\sqrt{1+\frac{8q^2}{g y_1^2}}\right]$$