1. Show the Gauss-Markoff model with outliers. Show a hypothesis testing to detect outliers in the model and the potential problems in the testing. (15%)

2. What is the possible method for estimation of variance and covariance components? Show an example of the application of covariance estimation. (15%)

3. iBeacon is a technology for indoor localization via Bluetooth Low Energy (BLE). The observation of iBeacon is the signal between receiver (e.g. mobile phone) and broadcaster (i.e. beacon device). As the signal strength is related to distance, we can use multiple beacons to determine the position of mobile device for indoor navigation. Please answer the following questions:

   (a) In figure 1a, assume that the positions of beacons are error free and the coordinates of beacons are (X1,Y1), (X2,Y2), (X3,Y3) and (X4,Y4). The observations and standard errors of distance are R1±σ1, R2±σ2, R3±σ3 and R4±σ4. Please provide a procedure to calculate the user position (X,Y) using least squares adjustment. (15%)

   (b) Please analyze and sketch the error ellipse of an unknown point C in figures 1b and 1c, respectively. How to improve the configuration of figures 1b and 1c by adding an additional beacon D ? (15%)
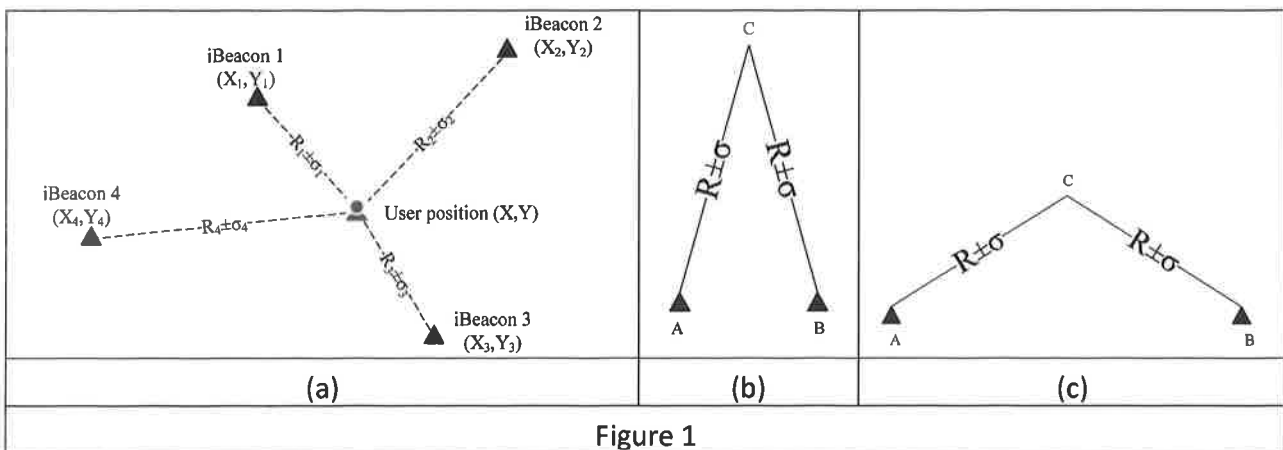
Figure 1

4. As shown in Figure 2, A, B are stations with known coordinates, while P, Q are stations of which coordinates to be determined. Four angles are measured from stations P, Q.

(a) Please provide two different computation schemes for the coordinates of P and Q (20%).

(b) Provide the equation for estimating the uncertainties propagated from the observations to the coordinates, assuming that the coordinates of A and B are error free (20%).

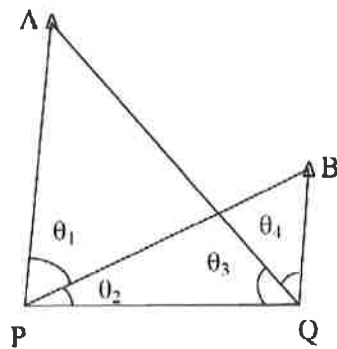Please provide the detail equations for the computation and their derivation.

(Total mark: 40%)

Figure 2

1. (25%) A potential V can be represented by the spherical harmonic series

$$V(r,\theta,\lambda)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left(\frac{R}{r}\right)^{n+1}\bar{v}_{nm}\bar{Y}_{nm}(\theta,\lambda)$$

The boundary value $\bar{v}_{nm}$ (in complex) have been given on the level surface where $r=R$.

a) Elaborate on the potential V where height$=(R+h), h>0$

b) What are the harmonic coefficients of $\dfrac{\partial V}{\partial r}$ if it is given on the level surface where r$=R$?

c) What are the harmonic coefficients of $\dfrac{\partial^2 V}{\partial r^2}$ if it is given on the level surface where r$=R$?

2. (25%) The geoidal undulation can be determined by Stokes' integral

$$N=\frac{R}{4\pi\gamma}\iint_{\sigma}\Delta g S(\psi)d\sigma$$

Suppose that the semi-major axis a' of the new reference ellipsoid is revised from the axis a by $a'=a+\Delta a$, and all other parameters remain the same

    a)   Elaborate if the normal gravity is affected

    b)   Elaborate if the geoidal undulation determined by Stokes' integral is affected when the new reference ellipsoid is adopted

3.　(25%) Consider a homogeneous spherical shell of mass M, $GM = 4 \times 10^{10} m^3 / s^2$. The inner and outer radii a and b of the shell are 3000km and 4000km respectively. Suppose P is a point at distance r, r>a. Derive $\partial V / \partial r$ at point P.
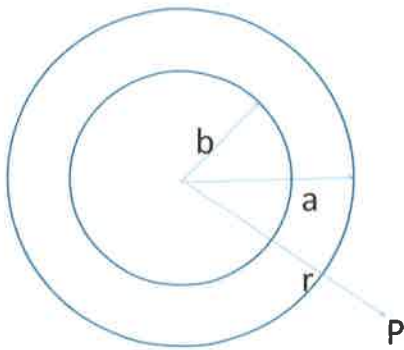
4　(25%) The realization of geoid can be achieved by either gravimetric approach or geometric approach. Briefly explain these two approaches (address on what kind of geodetic data are required, for example)

1.　Answer the following questions about satellite radar altimetry.

(a) How does a radar altimeter measure the distance between the satellite and the Earth's surface? What is a radar waveform? (10%)

(b) List typical error sources in measuring surface heights by a radar altimeter (10%)

(c) How are altimeter measurements used to determine gravity anomalies in the oceans? (5%)

(d) How are altimeter measurements used to determine ocean circulations? (5%)

2.　Answer the following questions about satellite gravity mission GRACE.

(a)　How does GRACE determine gravity? (10%)

(b)　GRACE gives monthly or weekly solutions of gravity. Typically, the gravity solutions are expressed in the following spherical harmonic expansion:

$$V(r,\theta,\lambda) = \frac{GM}{r}\sum_{n=0}^{N}(\frac{a}{r})^{n}\sum_{m=0}^{n}(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda)\overline{P}_{nm}(\cos\theta)$$

What is the relation between N and the shortest resolving wavelength in a gravity solution? (5%)

(c)　How is mass change derived from gravity change? What are the potential methods for improving the spatial resolution of mass change from GRACE? (5%)