

1. (25%) As shown in Fig.1, a uniform beam of length L has one end fixed and the other end attached a rigid mass (M) with a spring having stiffness k . Apply the simple beam theory to obtain the characteristic equation for determining the natural frequencies of the beam under transverse vibration if the rigid mass only moves in the transverse direction. The beam has cross-sectional area A , mass per unit volume ρ , area moment of inertia I and modulus of elasticity E .

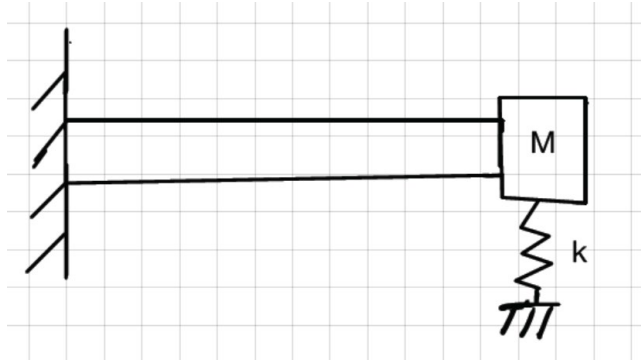


Fig. 1

2. Consider the equations of motion for an N-degree-of-freedom system, $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}$, where \mathbf{M} , \mathbf{C} and \mathbf{K} are symmetric; \mathbf{M} is also positive definite.
- (a) (5%) If one wants to apply the technique of real mode decomposition to obtain the responses, what condition must \mathbf{M} , \mathbf{C} and \mathbf{K} satisfy?
- (b) (20%) Formulate the solution for the above equations of motion with zero initial conditions via the technique of real mode decomposition. Assume the system has no repeated natural frequencies.
3. (25%) A SDOF system with mass $m=80,000$ kg. It is under free vibration. Initial displacement is $u_0 = 30$ mm. Initial velocity is $\dot{u}_0 = 0$. The amplitude is 20mm after the first cycle. The period is 0.5sec. Please calculate the stiffness, damping ratio and damping coefficient of the system.
4. (25%) What is the response spectrum? Given a ground acceleration, how do you generate an acceleration spectrum?